A DISCRETE TIME MODEL OF DURABLE GOODS WITH QUALITY UNCERTAINTY AND THE ROLE OF WARRANTY

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1. Introduction

Durable goods are often traded in the secondhand market a couple of years after they are purchased as new goods. The quality of durable goods can be defined by various kinds of measure such as durability and maintenance (or repair) cost associated with its usage. Recently, secondhand markets have grown around the world. It is expected that, especially for automobiles, the amount of traded secondhand goods will increase in the future.

In an actual economy, traded secondhand goods have various qualities. The prices, of course, vary depending on their quality. In addition, the original quality of secondhand good is unknown when it is purchased new. Usually, producers of new goods assure the quality of new goods. Alternatively, instead of producers, an insurance company can offer various kinds of insurance services for new goods. A typical insurance policy offered by an insurance company is described as follows: receiving the premium from the consumer, the insurance company compensates the consumer, if the good gets out of order within a certain period of time. When the insurance service is offered by producers, we usually call it a warranty. Then the insurance premium can be considered to be included in the price of new goods.

Consumers have several options for durable goods. For example, each consumer can not only keep his or her durable good but also sell it and buy a new one. Or the consumer can scrap it and buy a new one or another secondhand good. Each consumer also decides how much he or she borrows or saves when purchasing a durable good, since durable goods are generally expensive. How much the consumer borrows or saves depends on the former decision, namely whether he or she decides to own a new good or a secondhand one.

Thus consumer’s decision is complicated. Under a warranty service, each consumer decides

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each period 1) whether he or she should own a new or a secondhand good, and 2) how much he or she should borrow or save, so that his/her utility can be maximized.

The main purpose of this paper is to analyze consumer’s behavior in four competitive markets, namely new-good market, secondhand-good market, capital market, and labor market. The second purpose is to investigate the role of warranty for the quality of durable goods and to characterize the efficient design for a warranty. As an example of durable goods, we consider automobiles. Formally, in a discrete-time general equilibrium model we try to answer these questions: how the warranty service can be designed, how each consumer chooses either a new good or a secondhand good, how much each consumer borrows or saves, the quantity of new or secondhand goods in the whole economy each period, and whether the competitive equilibrium can attain the social optimum as defined later.

Durable goods are, however, inclined to be produced by large companies affecting the market prices. Therefore, it may not be appropriate to assume perfect competition in our model. Because of several reasons, however, durable-goods markets have become more competitive. First, generally, world trade has expanded for several decades. For example, exports (or imports) of automotive products in 2001 are quadruple what they were in 1980. Although Japan has been one of the leading exporters of automobiles, recently, the ratio of imported automobiles to domestic ones in Japan has nearly doubled in a decade. Secondly, secondhand markets have grown around the world. Since secondhand goods are important substitutes for newly produced goods, this tendency seems to relax the oligopolistic structure of automobile industry. Indeed, in the United States, the ratio of used automobiles sales to total automobiles sales has increased from 60% to 71% in a decade. The ratio in Japan has also increased from 52% to 61%.

In this paper, the main focus is not on the analysis of producer’s decisions but on the analysis of consumer’s behavior in the new-good market, secondhand-good market, labor market, and capital market. Throughout this paper, for simplicity, we assume competitive markets.

Most previous studies concerning the durable goods restricted their attention to the producer’s decisions; especially, the choice of durability by a monopoly seller. In his pioneering studies, Swan (1970, 1971) concluded that a monopoly seller will produce output whose durability is equivalent to the socially optimal level. Some researchers have attempted to reexamine Swan’s result by relaxing some of the key assumptions in his model (see Parks

Recently, however, some interesting studies shed light on consumers’ decisions associated with durable goods (see Rust (1985), Mann (1992), Anderson and Ginsburgh (1994), Waldman (1996), Kinokuni (1999), Hendel and Lizzieri (1999a, 1999b), and Konishi and Sandfort (2002)). They incorporate in their models an active secondhand-good market, in which each consumer can trade secondhand goods. Although most of the authors consider the planned obsolescence practiced by a monopolist, Rust (1985), Hendel and Lizzieri (1999b), and Konishi and Sandfort (2002) take a different approach. These latter studies analyze consumers’ decisions assuming a competitive market.

These investigations also consider consumers’ decisions under the uncertainty associated with secondhand-good quality (see Rust (1985), Hendel and Lizzieri (1999a, 1999b), and Konishi and Sandfort (2002)). However, they analyze the steady-state, rational-expectation equilibrium. Assuming this kind of equilibrium, as Rust (1985) proved, Pareto efficiency can be obtained by the competitive market solution. Clearly, it is not necessary to introduce any warranty system for durable goods if the economy is always in a steady-state, rational-expectation equilibrium. In addition, it is assumed in their models that consumers can not save or borrow.

In this paper, we do not assume the steady-state, rational-expectation equilibrium. Therefore, we need a mechanism such as the warranty system that enables each consumer to hedge the risk associated with the secondhand-good quality. We also assume that each consumer can borrow or save in the competitive capital market. The warranty system should be determined endogenously by market clearance conditions. In our model, we consider four markets, namely new-good market, secondhand-good market, capital market, and labor market.

Instead of assuming the steady-state, rational-expectation equilibrium, we have to accept several restrictive assumptions to obtain our main results analytically. First, Rust (1985), Anderson and Ginsburgh (1994), Hendel and Lizzieri (1999a, 1999b), and Konishi and Sandfort (2002) assume the heterogeneous consumers’ preferences. Rust (1985) also analyzes the model with homogeneous consumers, and shows that each consumer is indifferent to all options available each period. This means that a homogeneous consumer model cannot provide a positive theory for the existence of secondhand-good market. In this paper, however, we focus on the financial aspects of consumers’ decisions and the role of warranty in the durable-goods markets. With this as our rationale, for simplicity, we shall assume homogeneous consumers’
preferences. The effect of the heterogeneity of consumers' preferences on our results will be discussed later.

Secondly, we shall ignore the problem of asymmetric information represented by Akerlof's (1970) lemons problem, although there is a vast literature on it (see Dionne and Doherty (1992) for a survey). Unfortunately, the empirical evidence on the presence of adverse selection is inconclusive (see Bond (1982) and Genesove (1993)). This is one of the reasons why we shall ignore asymmetric information problem.

The outline of this paper is as follows. In Section 2, we consider the social optimum problem as a benchmark. In Section 3, we characterize the competitive market structure and analyze consumer's behavior in the competitive markets. Section 4 presents concluding remarks.

II. The Social Optimum

We consider a society in which each consumer is assumed to possess one durable good each period. As an example of durable goods, we consider automobiles which can be used at most for two periods. All used cars are qualitatively not equivalent to each other. They can be distinguished by their repair costs which should be paid in the second period.

A car in its first period is called a new one, and clearly no repair is required before it is used. On the other hand, a car is called an old one in the second period and repair is assumed to be required before it is used. Furthermore, the repair cost, denoted by \( m \), for an old car is assumed to be unknown when it is purchased as a new car (namely, in the first period). What is known, in the first period, is the distribution function of repair cost \( m \), which is represented by \( F: [0, \bar{m}] \rightarrow [0,1] \). We assume that \( F \) is strictly increasing and has density \( f \). We assume that we know the true \( m \) for each car in the second period. When the repair cost \( m \) is very high, it can be disposed of instead of being used as an old car in the second period.

Each consumer is endowed with a fixed amount of leisure denoted by \( L \) each period. We consider it as numeraire. The utility of each consumer depends on leisure each period. If a consumer sacrifices an amount of leisure \( q_i \) for labor in period \( i \), his or her utility is represented by

\[
U(L - q_0, L - q_1, \ldots, L - q_T).
\]
We assume that $U_u < 0$ and $U_y = 0$ ($i \neq j$), where $U_u$ and $U_y$ denote partial derivatives with respect to $(L - q_t)$.

We assume a fixed number of consumers, $n$, in the society. Denote by $n_i^1$ the number of consumers with a new car in period $i$. Then, $(n - n_i^1)$ consumers use old cars. Note that no old cars can exist in the initial period ($t = 0$) and the society is constrained by $n_0^1 = n$.

We assume constant returns to scale for production of a new car. Denote by $p$ the constant average cost of a new car. Denoting by $M_i$ the maximum value of repair cost in period $i$, note that all old cars available in the period have the corresponding repair costs lower than $M_i$. Then, the expected repair cost for an old car, in period $i$, can be represented by

$$
\mu_i = \frac{\int_0^{M_i} mF(m) \, dm}{F(M_i)}.
$$

We assume that the social welfare is defined as the sum of consumers’ utilities. Therefore, the social optimum problem can be formulated as follows:

$$
\text{Max} \quad nU(L - q_0, L - q_1, \ldots, L - q_T)
$$

subject to

$$
n q_i = (n - n_i^1) \mu_i + p n_i^1 \quad i = 1, \ldots, T
\tag{1}
$$

$$
n_i^1, F(M_i) = n - n_i^1 \quad i = 1, \ldots, T
\tag{2}
$$

where

$$
n_0^1 = n
$$

$$
q_0 = p
\tag{3}
$$

The left-hand side of (1) represents the total labor supply in the society and the right-hand side represents the sum of labor input in the repair sector and in the automobile producing sector. On the other hand, suppose that $n_{i-1}^1$ consumers possess new cars in period $i-1$. In the next period, of $n_i^1$ cars, those with repair cost lower than $M_i$ can be reused as old cars. Then, equation (2) can be considered as a kind of the accounting identity for the vehicle stock.

The Lagrangian of the problem can be written as

$$
\Gamma = nU(L - q_0, L - q_1, \ldots, L - q_T) + \sum_{i=1}^{T} \lambda_i (n q_i - (n - n_i^1) \mu_i - p n_i^1) + \sum_{i=1}^{T} \psi_i (n - n_i^1 - n_{i-1}^1, F(M_i))
$$.
where $\lambda$ and $\psi$ denote Lagrange multipliers associated with (1) and (2) respectively. First order conditions for the optimum can be written as

$$\frac{\partial \Gamma}{\partial q_i} = -nU_i + n\hat{\lambda}_i = 0 \quad i = 1, \cdots, T \tag{4}$$

$$\frac{\partial \Gamma}{\partial n_i} = \lambda_i (\mu_i - p) - \psi_i - \psi_{i+1} F(M_{i+1}) = 0 \quad i = 1, \cdots, T - 1 \tag{5}$$

$$\frac{\partial \Gamma}{\partial n_i} = \lambda_i (\mu_i - p) - \psi_i = 0 \tag{6}$$

$$\frac{\partial \Gamma}{\partial M_i} = -\lambda_i (n - n_i) \frac{\partial \mu_i}{\partial M_i} - \psi_i n_i f(M_i) = 0 \quad i = 1, \cdots, T \tag{7}$$

where $u_i$ denotes the partial derivative of $U$ with respect to $L - q_i$. Note that $\frac{\partial \mu_i}{\partial M_i}$ can be calculated as

$$\frac{\partial \mu_i}{\partial M_i} = \frac{f(M_i)}{F(M_i)} (M_i - \mu_i) \quad i = 1, \cdots, T \tag{8}.$$

Substituting (8) into (7) and using (2), we can rewrite (7) as

$$\psi_i = -\hat{\lambda}_i (M_i - \mu_i) \quad i = 1, \cdots, T - 1 \tag{9}.$$

Comparing (6) with (9) in the last period $T$, we can obtain

$$M_T = p \tag{10}.$$

Equation (5) can be rewritten as

$$\frac{U_{i+1}}{U_i} = \frac{p - M_i}{(M_{i+1} - \mu_{i+1}) F(M_{i+1})} \quad i = 1, \cdots, T - 1 \tag{11}.$$}

using (4) and (9).

To interpret (11), it is convenient to rewrite it as follows:

$$U_i p - U_{i+1} F(M_{i+1}) \mu_{i+1} = U_i M_i + U_{i+1} F(M_{i+1}) M_{i+1}.$$

Now suppose that the number of new cars $n'_i$ in period $i$ increases marginally, that is from $n_i'$ to $n'_i + 1$. The society should pay $p$ for an additional new car in period $i$. The additional new car can be used as an old car in period $i+1$ if the repair cost $m$ is smaller than $M_{i+1}$. When this event occurs, the society should pay the repair cost $m_{i+1}$ for the car in period $i+1$. Note
that the expected repair cost for the car in period $i+1$ is $F(M_{i+1})\mu_{i+1}$. Then the left hand side of (12) represents the marginal loss of social welfare (measured by utility of the representative consumer) by increasing the number of new cars by one in period $i$.

On the other hand, the society can obtain some gains by marginally increasing the number of new cars in period $i$. First, the society can abandon the old car with the highest repair cost and save the repair cost $M_i$. Secondly, if the repair cost of the additional new car is lower than the highest repair cost $M_{i+1}$, the society can abandon the old car with $M_{i+1}$ and save the repair cost $M_{i+1}$. Note that the probability that the event occurs is represented by $F(M_{i+1})$. After all, the right hand side of (12) represents the marginal gains from increasing the number of new cars in period $i$. Therefore, according to (12), each period society should set the number of new cars at the level where the marginal gain from increasing new cars by one is equal to the marginal loss.

III. The Competitive Framework

In this section, we characterize the competitive market equilibrium. Can the competitive market attain the social optimum presented in the previous section? We will see that the answer for this question is yes. Simultaneously, we analyze consumer's behavior in the competitive market. First, we introduce, in the competitive market, a used-car market and a warranty service for repair cost of an old car. In the first subsection, we derive the warranty design under competitive equilibrium. It can be characterized as a full-coverage insurance with an upper limit (Proposition 1).

Under this warranty service, consumer’s behavior will be analyzed in the following subsections. Each consumer decides each period 1) whether he or she owns a new or an old car, and 2) how much he or she borrows or saves, so that the utility can be maximized. In the second subsection, we derive Proposition 2 which shows that each consumer is indifferent to the choice between a new car and an old car each period. This proposition also shows that each consumer enjoys the same amount of leisure each period. We also analyze the effect of lack of the warranty service with full coverage on the price of an old car (Proposition 3).

In the beginning of the last subsection, we derive Proposition 4, which shows how much each consumer in period $i$ borrows or saves at endogenously determined interest rate. Then, we examine the remaining competitive equilibrium conditions: i) the accounting identity for the
vehicle stocks, including the used-car market clearance condition, and ii) the market clearance conditions for the capital market and the labor market.

Gathering all of the competitive equilibrium conditions, we can conclude that the social optimum can be attained in the competitive market.

**Used car market and warranty system design.** We begin by introducing a used (or old) car market. Each old car can be differentiated by its repair cost. The market price of an old car reflects its quality measured by the repair cost. Denote by \( K_i \) the price of the old car with the highest quality or with zero repair cost in period \( i \). Then it can be shown that the price of old cars with repair cost \( m' \) in period \( i \), denoted by \( P^u_i(m') \), is given by

\[
P^u_i(m') = K_i - m'.
\]  

(13)

This equation follows from consumer's arbitrage behavior. Note that \( K_i \) is determined endogenously in the competitive market.

Assume that the following warranty service for a new car is available in the competitive economy. When a consumer buys a new car in period \( i \) \((i = 0, \cdots, T-1)\), he or she is required to pay the premium \( w_i \) to the producer of the new car. In period \( i + 1 \), the repair cost of the car is known and the producer compensates the consumer who purchased it as a new car in period \( i \).

The compensation from the producer can be represented by

\[
x_{i+1} m_{i+1} \text{ if } m_{i+1} \leq K_{i+1} \\
x_{i+1} K_{i+1} \text{ if } m_{i+1} > K_{i+1},
\]  

(14)

where \( x_{i+1} \) denotes the rate of coverage \((0 < x_i \leq 1)\). To see that the repair cost to be compensated is at most \( K_{i+1} \), note that the producer can compensate the consumer with an old car whose quality is the highest, instead of paying him or her the repair cost higher than \( K_{i+1} \).

In the competitive equilibrium, the premium \( w_i \) should be fair in the following sense. The premium paid by a consumer should be equal to the discounted expected compensation paid by a producer to the consumer. This corresponds to the competitive equilibrium condition that the expected profit obtained by the warranty service is zero. Therefore, the premium \( w_i \) can be calculated as

\[
w_i = x_{i+1} \mathcal{E}_{i+1} (E_{K_{i+1} \geq m_{i+1}} [m_{i+1}] - E_{K_{i+1} < m_{i+1}} [K_{i+1}]).
\]  

(15)
where \( E[] \) denotes expectation represented by
\[
E_{K_{i+1} \geq m_{i+1}} [m_{i+1}] = \int_0^{K_{i+1}} m dF(m),
\]
\[
E_{K_{i+1} < m_{i+1}} [K_{i+1}] = \int_{K_{i+1}}^{\infty} K_{i+1} dF(m).
\]

Note that \( r_{i+1} \) denotes the interest rate (\( \delta_{i+1} = \frac{1}{1 + r_{i+1}} \)), and that both the interest rate \( r_{i+1} \) and the discount factor \( \delta_{i+1} \) should be determined endogenously.

When the warranty service characterized by (14) and (15) is available, the discounted expected expenditure of a consumer who buys a new car in period \( i \) can be represented by
\[
p + w_i + \delta_{i+1} (1 - x_{i+1})(E_{K_{i+1} \geq m_{i+1}} [m_{i+1}] + E_{K_{i+1} < m_{i+1}} [K_{i+1}]).
\]

Because of (15), it can be seen that the above expression does not depend on \( x_{i+1} \). However, the variance of the expenditure in period \( i+1 \) depends on \( x_{i+1} \). Specifically, we can exclude the volatility of it, if the rate of compensation is set at one (that is, \( x_{i+1} = 1 \)). By assumption that each consumer is risk averse (formally, represented by \( U_i < 0 \) and \( U_j = 0 \) (\( i \neq j \)), the utility of the consumer can be maximized at \( x_{i+1} = 1 \). Then, the consumer prefers full coverage, namely \( x_{i+1} = 1 \) (\( i = 0, \ldots, T - 1 \)). Assuming risk neutrality of producers, they are indifferent among \( x_{i+1} \). If the number of consumers \( n \) is very large, the law of large numbers implies that the risk neutrality assumption for producers is relevant. Therefore, the warranty design in the competitive economy can be characterized as full coverage. These results can be summarized as Proposition 1.

**Proposition 1.** The warranty design in the competitive market can be characterized as follows. The premium \( w_i \) is represented by
\[
w_i = \delta_{i+1}(E_{K_{i+1} \geq m_{i+1}} [m_{i+1}] + E_{K_{i+1} < m_{i+1}} [K_{i+1}]).
\]
(16)
The compensation from the producers is represented as
\[
m_{i+1} \text{ if } m_{i+1} \leq K_{i+1},
\]
\[
K_{i+1} \text{ if } m_{i+1} > K_{i+1}.
\]
(17)
The warranty design represented by (16) and (17) is referred to as an insurance policy with an upper limit on full coverage in insurance economics (see Raviv (1979)). It is well known
that full-coverage insurance policy is Pareto optimal (see Mossin (1968) and Shavell (1979) for examples). In addition, Raviv (1979) shows that Pareto optimal insurance policy does not involve an upper limit on coverage. He explains the prevalence of upper limits on coverage which are frequently incorporated in major medical, liability, and property insurance as follows. His explanation rests on the fact that insurance companies are required to sell a policy with a prescribed actuarial value. We offer an alternative explanation for the existence of upper limits on full coverage. Note that the upper limit in our model is given by the price of the used car with the highest quality. That is, there exists an upper limit on compensation due to the existence of the perfect substitutes (namely, used cars) and their market (or used-car market).

Before analyzing consumer’s behavior further, we briefly discuss the effect of the heterogeneity of consumers’ preferences. The main question is whether the uniform warranty service characterized by the previous proposition can still be efficient even if consumers’ preferences are not homogeneous. The answer is yes, since any uncertainty vanishes if the warranty service is provided. This role of warranty can not be affected by the heterogeneity of consumers’ preferences. Therefore, Proposition 1 is still valid in the presence of the heterogeneous consumers.

**Consumer’s behavior in the competitive markets.** The consumer has to decide whether he or she owns a new car or an old one each period. Suppose that the time series representation of a new or an old car such as

\[
(new_0, \cdots, new_{i-1}, old_i, new_{i+1}, \cdots)
\]

means that a consumer owns an old car in period \(i\) and so on. Hereafter, we call this the plan of the consumer. It can be shown that each consumer is indifferent to a choice among all plans.

To see this, without any loss of generality, it is sufficient to compare the following two plans: \((new_0, \cdots, old_{i-1}, new_i, old_{i+1}, \cdots)\) and \((new_0, \cdots, old_{i-1}, old_i, old_{i+1}, \cdots)\). Note that the two plans are different only in period \(i\). The discounted (in period \(i\)) expected expenditure in period \(i\) and \(i+1\), along the two plans, can be calculated as \(p + w_i + K_i + \delta_{i+1} K_{i+1}\) respectively.

Suppose that \(p + w_i < K_i + \delta_{i+1} K_{i+1}\). Then, no one chooses the latter plan. The same conclusion can be obtained by comparing two other plans such as \((new_0, \cdots, new_{i-1}, new_i, old_{i+1}, \cdots)\) and \((new_0, \cdots, new_{i-1}, old_i, old_{i+1}, \cdots)\). Therefore, if \(p + w_i < K_i + \delta_{i+1} K_{i+1}\), no one chooses an old car in period \(i\). Conversely, if \(p + w_i > K_i + \delta_{i+1} K_{i+1}\), no new car is purchased in that period.
Both of these cases can not be consistent with the competitive equilibrium. Therefore, the following equation must be satisfied:

\[ p + w_i = K_i + \delta_{i+1} K_{i+1}. \]  

(18)

It can be easily shown that the sum of the discounted expenditure is equivalent along all plans, if equation (18) is satisfied for all periods.

In addition to the choice between a new car and an old car, each consumer can decide how much he or she borrows or saves each period. Denote by \( s_i \) the borrowing of a consumer in period \( i \). If \( s_i \) is negative, the consumer saves money. By choosing \( s_i \) optimally, each consumer can allocate the sum of the discounted expenditure so that his or her utility can be maximized. Note that the sum of the discounted expenditure is equivalent along all plans. Therefore, the maximized utility is also equivalent among all consumers (along all their plans) if each \( s_i \) \( (i = 1, \ldots, T) \) is chosen optimally. Formally, \( s_i \) must satisfy

\[ \frac{U_{i+1}}{U_i} = \delta_{i+1} \quad i = 0, \ldots, T - 1. \]  

(19)

where \( U_i \) and \( U_{i+1} \) denote partial derivatives. These results show that each consumer is indifferent to a choice among all plans. In addition, assuming that the solution for \( s_i \) is unique for each consumer at the equilibrium, each consumer enjoys the same amount of leisure or the same level of utility each period. These results can be summarized as a proposition.

**Proposition 2.** In the competitive equilibrium, each consumer is indifferent to a choice among all plans. Moreover, assuming uniqueness of the solution for the optimal borrowing problem, each consumer enjoys the same level of utility each period.

This proposition is consistent with the result obtained in Rust (1985). He correctly argues that with homogeneous consumers, all durable goods must be priced to yield the same level of utility and that consumers are indifferent to the choice of the durable good. *Proposition 2* shows that his statement is still correct even if the economy is not in a steady-state, rational-expectation equilibrium but in an ordinary general equilibrium. However, as Rust (1985) and Hendel and Lizzeri (1999b) show, the proposition can not be valid in the presence of heterogeneous consumers. Under the assumption of heterogeneity of consumers’
preferences, each consumer is not indifferent to choice among all options (e.g. one option is represented by selling an old car and buying a new car) and prefers one particular option to other options in every period. Which option a consumer chooses in every period depends on his or her preference. Since we do not assume any steady-state, rational-expectation equilibrium, it is not easy to derive the efficient plan for each heterogeneous consumer analytically.

Before we argue the competitive equilibrium conditions further, we shall simply compare the competitive outcomes with and without the warranty. The next proposition shows that the price of an old car will be increased if it were not for the full coverage warranty service.

**Proposition 3.** If the warranty service is offered without full coverage, the price of an old car will be increased.

**Proof.** Under the full coverage warranty service, we obtain (18) each period. It is convenient to rewrite equation (18) using (16) and (19) as

\[ pU_i + (F(K_{i+1})\phi_{i+1} + (1 - F(K_{i+1}))K_{i+1})U_{i+1} = K_iU_i + K_{i+1}U_{i+1}, \]

(20)

where

\[ \phi_{i+1} = \int_0^{K_{i+1}} \frac{mdF(m)}{F(K_{i+1})}. \]

As discussed above, if the warranty service is not available with full coverage of repair costs (i.e. if \( x_i < 1 \)), there still remains uncertainty associated with purchasing a new car. In this case, no one will purchase a new car, if the discounted expected expenditures are equivalent along all plans. Therefore, they should vary along each plan in the competitive equilibrium, if the warranty service with full coverage is not available. A consumer who chooses an old car should pay more than the one who purchases a new car. Formally, if it were not for the full coverage warranty service, the following inequality should be satisfied at the equilibrium:

\[ p + w_i + \delta_{i+1}(1 - x_{i+1}) (E_{K_{i+1} \geq m_{i+1}}[m_{i+1}] + E_{K_{i+1} < m_{i+1}}[K_{i+1}]) < K_i + \delta_{i+1}K_{i+1}. \]

We can also rewrite this inequality using (15) and (19) as

\[ pU_i + (F(K_{i+1})\phi_{i+1} + (1 - F(K_{i+1}))K_{i+1})U_{i+1} < K_iU_i + K_{i+1}U_{i+1}. \]

(21)
By comparing (21) and (20), we can infer the effect of insufficient warranty service on the price of an old car. The lack of the warranty service with full coverage leads to an upward pressure on the price of an old car. To see this, consider a consumer who purchases an old car in every period except in the initial period. Now suppose that equation (20) is still valid. Then the equation (20) applied to the consumer can be rewritten as

\[ pU_i(\cdots, L - K_{i}, L - K_{i_{1}}, \cdots) + (F(K_{i_{1}})\phi_{i_{1}} + (1 - F(K_{i_{1}}))K_{i_{1}})U_{i_{1}}(\cdots, L - K_{i}, L - K_{i_{1}}, \cdots) \]

\[ = K_{i}U_i(\cdots, L - K_{i}, L - K_{i_{1}}, \cdots) + K_{i_{1}}U_{i_{1}}(\cdots, L - K_{i}, L - K_{i_{1}}, \cdots). \]

Differentiating both sides of the equation with respect to \( K_{i_{1}} \), we obtain

\[ \frac{\partial l.h.s.}{\partial K_{i_{1}}} = (1 - F(K_{i_{1}}))U_{i_{1}} - (F(K_{i_{1}})\phi_{i_{1}} + (1 - F(K_{i_{1}}))K_{i_{1}})U_{i_{1}} + \]

\[ \frac{\partial r.h.s.}{\partial K_{i_{1}}} = U_{i+1} - K_{i+1}U_{i+1}. \]

Note that \( U_{ij} = 0 \ (i \neq j) \). We can easily see that \( \frac{\partial l.h.s.}{\partial K_{i_{1}}} < \frac{\partial r.h.s.}{\partial K_{i_{1}}} \). In fact, the inequality (21) should be satisfied for the consumer. Therefore, it follows that \( K_{i_{1}} \) will be increased if it were not for the full coverage warranty service. \( Q.E.D. \)

The interpretation of Proposition 3 is as follows: if the compensation is not sufficient, a risk-averse consumer prefers an old car to a new car. As a result, the demand for old cars increases. This preference for old cars leads to an upward pressure on the price of an old car.

Here we go back to the previous arguments associated with competitive equilibrium. To show that the social optimum solution characterized by (1), (2), (3), and (11) can be obtained in the competitive market, we have to see if these equations (1), (2), (3), and (11) can be derived in the competitive framework. First, we can easily show that equation (11) is straightforwardly obtained from (20). Secondly, in the next subsection, we will see that the remaining equations (1), (2), and (3) are satisfied at the competitive equilibrium.

The market clearance conditions for the used-car market, capital market, and labor market. In this subsection, we derive two additional equations under competitive equilibrium corresponding to the remaining social optimum conditions (1), (2), and (3). At the competitive equilibrium, the following conditions must also be satisfied: first, the accounting identity for
the vehicle stock, including the used-car market clearance condition, and secondly, the market clearance conditions for the capital market and the labor market.

*The accounting identity for the vehicle stock including the used-car market clearance condition.*

We use the following symbols for simplicity:

\( n_{i-1}^{11} \): the number of consumers buying new cars in period \( i-1 \) and continuing to keep them in period \( i \).

\( n_{i-1}^{12} \): the number of consumers buying new cars in period \( i-1 \) and selling them in the used-car market and buying new cars in period \( i \).

\( n_{i-1}^{13} \): the number of consumers buying new cars in period \( i-1 \) and selling them and buying another old cars in the used-car market in period \( i \).

\( n_{i-1}^{14} \): the number of consumers buying new cars in period \( i-1 \) and abandoning them (because of their relatively high repair cost) and buying new cars in period \( i \).

\( n_{i-1}^{15} \): the number of consumers buying new cars in period \( i-1 \) and abandoning the (because of their relatively high repair cost) and buying old cars in period \( i \).

\( n_{i-1}^{21} \): the number of consumers owning old cars in period \( i-1 \) and buying new cars in period \( i \).

\( n_{i-1}^{22} \): the number of consumers owning old cars in period \( i-1 \) and buying old cars in period \( i \).

At the equilibrium, the following equations must be satisfied:

\[
\begin{align*}
n_{i-1}^1 &= n_{i-1}^{11} + n_{i-1}^{12} + n_{i-1}^{13} + n_{i-1}^{14} + n_{i-1}^{15} \quad (22) \\
n_{i-1}^2 &= n_{i-1}^{21} + n_{i-1}^{22} \quad (23) \\
n_i^2 &= n_{i-1}^{11} + n_{i-1}^{13} + n_{i-1}^{15} + n_{i-1}^{22} \quad (24) \\
n_i^1 &= n_{i-1}^{12} + n_{i-1}^{14} + n_{i-1}^{21} \quad (25) \\
n_{i-1}^{12} + n_{i-1}^{13} + n_{i-1}^{15} + n_{i-1}^{22} \quad (26)
\end{align*}
\]

where we denote \( n_i^1 \) and \( n_i^2 \) the number of consumers owning new cars and old cars in period \( i \) respectively. The last equation (26) represents the market clearance condition for the used-car market.

It can be easily seen that, in period \( i \), an old car must be abandoned if the repair cost is higher than \( K \). Conversely, no one abandons an old car with the repair cost smaller than \( K \).
Therefore, there must exist \( n_{i-1}, F(K_i) \) old cars in period \( i \) at the competitive equilibrium, that is,

\[
n_i^2 = n_i - n_i^1 = n_{i-1} F(K_i). \tag{27}
\]

Note that equation (27) corresponds to (2) in the social optimum framework. Alternatively, the argument stated above can be summarized as the following two equations:

\[
n_{i-1}^{11} + n_{i-1}^{12} + n_{i-1}^{13} = F(K_i) n_{i-1}^1 \tag{28}
\]

\[
n_{i-1}^{14} + n_{i-1}^{15} = (1 - F(K_i)) n_{i-1}^1. \tag{29}
\]

**The market clearance for the capital market and the labor market.** Finally, we examine the capital market clearance condition and derive it from the labor market clearance condition as the Walrus' law. To do this, it is necessary to calculate both the sum of the individual borrowings and the sum of the profits of producers obtained through the warranty service.

First, note that in the competitive equilibrium, each consumer enjoys the same level of leisure or the same level of utility each period (see Proposition 2). It is convenient to represent the common level of leisure and the common level of utility by \( L - v_i \) and \( U(L - v_0, L - v_1, L - v_2, \ldots, L - v_T) \) respectively. Note that \( v_i \) represents labor supply per capita in period \( i \).

Then we can calculate how much each consumer borrows as follows. In the initial period, each consumer enjoys leisure equal to \( L - p - w_0 + s_0 \), since he or she buys a new car with payment of the premium \( w_0 \) and borrows \( s_0 \). At the equilibrium, however, \( L - p - w_0 + s_0 \) must be equal to \( L - v_0 \). Therefore, we obtain \( s_0 = p + w_0 - v_0 \). The sum of the individual borrowings in the initial period is

\[
n s_0 = n(p + w_0 - v_0) \tag{30}
\]

Generally, the borrowing of a consumer denoted by \( s_i \) \( (i \neq 0) \) depends on the decision of the consumer in the period, such as selling the old car and buying a new car in period \( i \), and so on. The following proposition represents how much each consumer borrows or saves in period \( i \) \( (i \neq 0) \).

**Proposition 4.** Denote by \( L - v_i \) the common leisure that each consumer enjoys in period \( i \). How much each consumer borrows in period \( i \) depends on his or her decisions in period
i and period as follows: Each of \( n_{i-1}^{11} + n_{i-1}^{13} + n_{i-1}^{15} \) consumers borrows \( s_i = -v_i \), each of \( n_{i-1}^{12} + n_{i-1}^{14} \) consumers borrows \( s_i = -K_i - v_i + p + w_i \), each of \( n_{i-1}^{21} \) consumers borrows \( s_i = -v_i + p + w_i \), and each of \( n_{i-1}^{22} \) consumers borrows \( s_i = K_i - v_i \).

**Proof.** See Appendix A.

Using Proposition 4, we can aggregate the individual borrowings in the whole of the economy. The total borrowings in the economy in period \( i \) (\( i \neq 0 \))

\[
= -(n_{i-1}^{11} + n_{i-1}^{13} + n_{i-1}^{15})v_i + n_{i-1}^{22}(K_i - v_i) + (n_{i-1}^{12} + n_{i-1}^{14})(p + w_i - v_i - K_i) + n_{i-1}^{21}(p + w_i - v_i).
\]

(31)

Secondly, to derive the capital market clearance condition, we have to calculate the sum of the profits (defined as the revenue minus the expenditure) obtained through the warranty service each period. In the initial period it is clearly equal to \( nw_0 \). The capital market clearance condition requires that the sum of the individual consumer’s borrowings be equal to the sum of the profits from warranty service each period. Noting (30), the capital market clearance requires

\[
v_0 = p
\]

in the initial period. This equation corresponds to (3) in the social optimum problem.

On the other hand, using (16), the sum of the profits from the warranty service in period \( i \) can also be easily calculated as

\[
n_i^1w_i - n_{i-1}^1(F(K_i)\phi_i + (1 - F(K_i))K_i) = n_i^1w_i - \frac{1}{\delta_i}n_{i-1}^1w_{i-1}.
\]

Noting (31), the capital market clearance condition in period \( i \) (\( i \neq 0 \)) can be written as

\[
-(n_{i-1}^{11} + n_{i-1}^{13} + n_{i-1}^{15})v_i + n_{i-1}^{22}(K_i - v_i) + (n_{i-1}^{12} + n_{i-1}^{14})(p + w_i - v_i - K_i) + n_{i-1}^{21}(p + w_i - v_i)
\]

\[
- n_i^1w_i + \frac{1}{\delta_i}n_{i-1}^1w_{i-1} = 0.
\]

(33)

Equation (33) can be simplified to

\[
- nv_i + n_i^1p + F(K_i)\phi_i n_{i-1}^1 = 0
\]

(34)

(see Appendix B for derivation).
Therefore, the capital market clearance in period $i \ (i \neq 0)$ requires

$$ mv_i = n_i^1 p + F(K_i)\hat{\phi} n_{i-1}^1. $$

(35)

This equation, however, is the market clearance condition for the labor market. Indeed, the left hand side of (35) represents the total labor supply and the right hand side represents total labor demand. The first term on the right hand side corresponds to labor demand from the new car industry and the second term corresponds to labor demand from the repair industry. Note that (35) represents Walrus’ law in our general equilibrium model. It should be also noted that equation (35) corresponds to (1) in the social optimum framework. In addition, using (27), we can rewrite (35) as

$$ v_i = \frac{(n - n_i^1)}{n} \hat{\phi} i + \frac{n_i^1}{n} p. $$

In other words, the equilibrium labor supply per capita is equal to a weighted average of the expected repair cost of an old car and the production cost of a new car.

Similarly, we can verify that the capital-market clearance condition is satisfied in the last period by noting that $K = p$ and the fact that the consumer buying a new car is not required to pay the premium in the last period.

Thus, we can obtain four important equations, (20), (27), (32), and (35) as the competitive equilibrium conditions. These four equations correspond to (11), (2), (3), and (1) respectively, which are the necessary conditions for the social optimum problem. It follows that

$$ v_i = q_i, $$

$$ K_i = M_i. $$

(36)

Therefore, we can conclude that the social optimum can be attained in the competitive market with the availability of warranty service for repair cost.

VI. Concluding Remarks

The durable good considered in our model has the following characteristics: 1) the durable good can be used at most for two periods, but 2) a consumer must pay repair cost if he or she uses it in the second period, however, 3) the repair cost paid in the second period is unknown in the first period. As an example of such durable goods, we consider automobiles. We analyze
consumer's behavior in the competitive markets. Assuming homogeneous consumers, each consumer is indifferent to whether he or she possesses a new or an old car under the conditions of competitive equilibrium. The optimal borrowing of a consumer in any period depends both upon whether the consumer owns a new car or an old car in that period and the previous period.

Simultaneously, we also consider how the competitive market can be designed. The competitive market must have several characteristics: 1) consumers can trade old cars in the used-car market, 2) consumers can decide how much to borrow and save at the interest rate which is endogenously determined, and 3) warranty service for new cars is available and offered with an upper limit on full coverage.

We give an alternative explanation for the prevalence of upper limits on coverage which are frequently incorporated in major medical, liability, and property insurance. Our results show that there exists an upper limit due to the existence of the used-car market. The lack of such an insurance service has an effect on the price of an old car. If the compensation is not full, the price of an old car will increase.

A car in our model can be available at most for two periods. In actual economy, however, an old car can be used for multiple periods. In this modified setting, our results have the following implication. Full warranty should be applied to all used cars with different ages. Recently, various kinds of warranty services have been offered for used cars by some car producers and by used car dealers, although their compensations are not sufficient. In Japan, many used car dealers provide warranty for used cars. The effective length of time for the warranty is usually one year. According to Hendel and Lizzeri (1999a), for several cars (Infiniti and Lexus) warranty coverage is extended to the used cars. Insufficient insurance coverage for used cars seems to be due to immaturity of used-car market. Our theoretical results indicate that the prices of used cars will be reduced and the used-car market grows as more complete insurance coverage becomes available for all used cars.

Appendix A

Proof of Proposition 4. First, each of \( n_{1,1}^{11} + n_{1,1}^{13} + n_{1,1}^{15} \) consumers owns a new car in period \( i-1 \). Each of \( n_{1,1}^{11} + n_{1,1}^{13} \) consumers is fully compensated by the producer for the repair cost \( m \) (note \( m \leq K_i \)). Each of \( n_{1,1}^{11} \) consumers enjoys leisure \( L + m - m + s_i = L + s_i \) in period \( i \). Since each of \( n_{1,1}^{13} \) consumers sells the original old car and buys another old car with the repair cost
\( m' \), the consumer enjoys leisure represented by \( L + m + (K_i - m) - (K_i - m') - m' + s_i = L + s_i \) in period \( i \). On the other hand, each of \( n_{i-1}^{i3} \) consumers is only partially compensated by the producer for the repair cost \( m \) (note \( m > K_i \)). The consumer abandons the original car and buys another old car with the repair cost \( m' \). Then he or she enjoys leisure \( L + K_i - (K_i - m') - m' + s_i = L + s_i \) in period \( i \). These imply that each of \( n_{i-1}^{i1} + n_{i-1}^{i3} + n_{i-1}^{i5} \) consumers borrows \( s_i = -v_i \) in the period.

Secondly, each of \( n_{i-1}^{i2} + n_{i-1}^{i4} \) consumers also owns a new car in period \( i - 1 \). Denote by \( m \) the corresponding repair cost. The producer fully compensates each of \( n_{i-1}^{i2} \) consumers for the repair cost \( m \) (note \( m \leq K_i \)). After the compensation, the consumer sells the car in the used-car market at the equilibrium price \( K_i - m \). Then he or she again buys a new car and pays \( p + w_i \). Finally, his or her leisure in period \( i \) can be calculated as \( L + m + (K_i - m) - p - w_i + s_i = L + K_i - p - w_i + s_i \). This implies \( s_i = -v_i + p + w_i \). On the other hand, the producer only partially compensates each of \( n_{i-1}^{i2} \) consumers for the repair cost \( m \) (note \( m > K_i \)). Then each of them abandons the original old car and buys a new car paying \( p + w_i \). As a result, he or she enjoys leisure \( L + K_i - p - w_i + s_i \). This implies \( s_i = -v_i + p + w_i \).

Thirdly, each of \( n_{i-1}^{i1} \) consumers owns an old car in period \( i - 1 \) and buys a new car in period \( i \). Therefore, the consumer’s borrowing in period \( i \) can be easily calculated as \( s_i = -v_i + p + w_i \). On the other hand, each of \( n_{i-1}^{i2} \) consumers buys an old car with the repair cost \( m \), which is sold at the equilibrium price of \( K_i - m \). Therefore, his or her leisure can be represented by \( L - (K_i - m) - m + s_i \). This implies \( s_i = K_i - v_i \). Q.E.D.

Appendix B

The derivation of equation (34). The left hand side of (33) can be rewritten as follows:

\[-(n_{i-1}^{i1} + n_{i-1}^{i3} + n_{i-1}^{i5})w_i + n_{i-1}^{i2}(K_i - v_i) + (n_{i-1}^{i2} + n_{i-1}^{i4})(p + w_i - v_i - K_i) + n_{i-1}^{i1}(p + w_i - v_i)\]

\[-n_i^1w_i + \frac{1}{\delta_i}n_{i-1}^{i1}w_{i-1}\]

\[= -(n_{i-1}^{i1} + n_{i-1}^{i3} + n_{i-1}^{i5} + n_{i-1}^{i2} + n_{i-1}^{i4} + n_{i-1}^{i5} + n_{i-1}^{i2} + n_{i-1}^{i4} + n_{i-1}^{i5})v_i + (n_{i-1}^{i2} - n_{i-1}^{i4} - n_{i-1}^{i5})K_i + (n_{i-1}^{i2} + n_{i-1}^{i4} + n_{i-1}^{i5})(p + w_i)\]

\[-n_i^1w_i + \frac{1}{\delta_i}n_{i-1}^{i1}w_{i-1}\]

\[= -nv_i + (n_{i-1}^{i2} - n_{i-1}^{i4} - n_{i-1}^{i5})K_i + n_i^1p + \frac{1}{\delta_i}n_{i-1}^{i1}w_{i-1}\]
\[
= -m_i + n_i^1 + F(K_i)\phi, n_i^1 + \left((n_i^{15} - n_i^{17} - n_i^{18}) + (1 - F(K_i))n_i^1\right)K_i
\]
\[
= -m_i + n_i^1 + F(K_i)\phi, n_i^1 + \left(-n_i^{15} - n_i^{14} + (1 - F(K_i))n_i^1\right)K_i
\]
\[
= -m_i + n_i^1 + F(K_i)\phi, n_i^1.
\]

The second equality follows from (22), (23), (25), and \( n_i^1 + n_i^2 = n \). The third equality follows from (16). The fourth equality follows from (26). The last equality follows from (29).

\[Q.E.D.\]

References


Japan Automobile Dealers Association "Automobile Statistical Data" (in Japanese) http://www.jada.or.jp


不確実性下での離散時間型耐久消費財モデルと中古品保証制度の役割

新 髙 隆 嘉

ここで分析される耐久消費財モデルは次の特徴をもつ。1）耐久消費財は最大2期間使用可能であり、2）中古品は故障する可能性があり、3）新品購入可能性（1期目）においては2期目に発生する修理費用は未知である。本研究では、このような耐久消費財モデルを用いて、競争市場での消費者行動を分析するとともに、市場における中古品に対する保証制度のあり方およびその役割を分析する。なお、分析は一般均衡の枠組みでなされる。

本研究では主として次の結果を得た。（1）均衡において各消費者は新車を保有するか中古車を保有するかについて無差別となる（命題2）。競争市場で実現する中古車保証制度は、保証に上限があるタイプとして特徴付けられる（命題1）。（2）（1）のような中古車保証制度が実現すれば、社会的最適な状態が競争市場の枠組みで実現しうる。取引費用あるいは情報の非対称性が存在するために中古車保証制度が存在しない場合、それが存在する場合と比較して均衡での中古車価格が上昇するという歪みが生じる（命題3）。